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PERIODIC SOLUTIONS OF FRACTIONAL LAPLACE EQUATIONS: LEAST PERIOD, AXIAL SYMMETRY AND LIMIT

Zhenping Feng — Zhuoran Du

Abstract. We are concerned with periodic solutions of the fractional Laplace equation

$$(-\partial_{xx})^s u(x) + F'(u(x)) = 0 \quad \text{in } \mathbb{R},$$

where $0 < s < 1$. The smooth function F is a double-well potential with wells at $+1$ and -1. We show that the value of least positive period is $2\pi \times (1/{-F''(0)})^{1/(2s)}$. The axial symmetry of odd periodic solutions is obtained by moving plane method. We also prove that odd periodic solutions $u_T(x)$ converge to a layer solution of the same equation as periods $T \to +\infty$.

1. Introduction

In this paper we are concerned with the following fractional Laplace equation

$$(-\partial_{xx})^s u(x) + F'(u(x)) = 0, \quad x \in \mathbb{R}, \tag{1.1}$$

where $(-\partial_{xx})^s$, $0 < s < 1$, denotes the usual fractional Laplacian. F is a smooth double-well potential satisfying

$$\begin{cases} F(1) = F(-1) = 0 < F(u) \quad \text{for all } -1 < u < 1, \\ F'(1) = F'(-1) = 0 \end{cases} \tag{1.2}$$

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