Topological **M**ethods in **N**onlinear **A**nalysis
Volume 60, No. 2, 2022, 601–632
DOI: 10.12775/TMNA.2022.021

A-PRIORI BOUND AND HÖLDER CONTINUITY OF SOLUTIONS TO DEGENERATE ELLIPTIC EQUATIONS WITH VARIABLE EXPONENTS

Ky Ho — Le Cong Nhan — Le Xuan Truong

Abstract. We investigate the boundedness and regularity of solutions to degenerate elliptic equations with variable exponents that are subject to the Dirichlet boundary condition. By employing the De Giorgi iteration, we obtain a-priori bounds and the Hölder continuity for solutions. As an application, we obtain the existence of infinitely many small solutions for a class of degenerate elliptic equations involving variable exponents.

1. Introduction and main results

1.1. Setting problem and motivations. In this paper, we investigate the boundedness and the regularity of solutions to the following problem:

$$\begin{cases} -\operatorname{div} \mathcal{A}(x,u,\nabla u) = \mathcal{B}(x,u,\nabla u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

2020 *Mathematics Subject Classification.* 35B45, 35B65, 35D30, 35J20, 35J60, 35J70.

Key words and phrases. $p(\cdot)$-Laplacian; weighted variable exponent Lebesgue–Sobolev spaces; a-priori bound; Hölder continuity; De Giorgi iteration; localization method.

The first and third authors are supported by University of Economics Ho Chi Minh City, Vietnam.

The second author is supported by Ho Chi Minh City University of Technology and Education, Vietnam.