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FOURTH-ORDER ELLIPTIC PROBLEMS INVOLVING CONCAVE-SUPERLINEAR NONLINEARITIES

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Abstract. The existence of solutions for a huge class of superlinear elliptic problems involving fourth-order elliptic problems defined on bounded domains under Navier boundary conditions is established. To this end we do not apply the well-known Ambrosetti–Rabinowitz condition. Instead, we assume that the nonlinear term is nonquadratic at infinity. Furthermore, the nonlinear term is a concave-superlinear function which can be indefinite in sign. In order to apply variational methods we employ some delicate arguments recovering some kind of compactness.

1. Introduction

In this work we consider the fourth-order elliptic problem

$$\begin{cases} \alpha\Delta^2 u + \beta\Delta u = a(x)|u|^{s-2}u + f(x,u) & \text{in } \Omega, \\ u = \Delta u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where $\Delta^2 = \Delta \circ \Delta$ is the biharmonic operator, $N > 4$, $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain. We also assume that $a \in L^\infty(\Omega)$ and $s \in (1,2)$ with $\alpha > 0$ and $\beta \in (-\infty, \alpha\lambda_1)$. The first eigenvalue for the linear problem $(-\Delta, H_0^1(\Omega))$ is

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