

## A NOTE ON LOCAL MINIMIZERS OF ENERGY ON COMPLETE MANIFOLDS

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**ABSTRACT.** In this paper, we study the geometric rigidity of complete Riemannian manifolds admitting local minimizers of energy functionals. More precisely, assuming the existence of a non-trivial local minimizer and under suitable assumptions, a Riemannian manifold under consideration must be a product manifold furnished with a warped metric. Secondly, under similar hypotheses, we deduce a geometrical splitting in the same fashion as in the Cheeger–Gromoll splitting theorem and we also get information about local minimizers.

### 1. Introduction

It is well-known that there are strong connections between embedded minimal hypersurfaces and the study of the famous Allen–Cahn equation

$$(1.1) \quad \Delta u + u - u^3 = 0,$$

which have arisen in the context of the gradient theory of phase transitions. We refer the reader to [6] for a detailed discussion about these analogies.

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