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STRATEGIES TO ANNIHILATE COINCIDENCES OF MAPS FROM TWO-COMPLEXES INTO THE CIRCLE

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Abstract. Given a pair of maps from a two-complex into the circle, for which there exists essential coincidence, we compare the efficiency of three strategies to annihilate all of them, via homotopy deformation. The strategies consist of attaching an arc to the circle in different ways: gluing the arc just by one of its endpoints; gluing the two endpoints to a same point of the circle (so obtaining the eight figure); and gluing the two endpoints to two different points of the circle (so obtaining the theta figure). We prove that the three strategies have the same effect in the matter of annihilate all coincidences, regardless of the domain of the maps. We also study the coincidence problem of maps from closed surfaces into the eight and theta figures, including those one that may not be factored through the circle.

1. Introduction

Let $f, g\colon K \to S^1$ be maps from a finite and connected two-dimensional CW complex (a *two-complex*, for short) into the circle. Suppose that the pair (f, g) may not be deformed (by homotopy) to a coincidence free pair.

In [2] we consider the problem of taking the natural embedding $\iota\colon S^1 \hookrightarrow \mathcal{W}$ of the circle S^1 into the *balloon-graph* $\mathcal{W} = S^1 \vee I$ (obtained by gluing one of the endpoints of the closed interval $I = [0, 1]$ to a point in the circle) and so asking if the pair $(\iota \circ f, \iota \circ g)$ can be deformed to a coincidence free pair. In other words, we ask if it is possible to annihilate all coincidences of the pair (f, g) after attaching a "tail" to the circle. We solve this problem in [2] for maps from the

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