CRITICAL POINTS OF A MEAN FIELD TYPE FUNCTIONAL
ON A CLOSED RIEMANN SURFACE

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Abstract. Let $(\Sigma, g)$ be a closed Riemann surface and $H^1(\Sigma)$ be the usual Sobolev space. For any real number $\rho$, we define a generalized mean field type functional $J_{\rho, \phi}: H^1(\Sigma) \to \mathbb{R}$ by

$$J_{\rho, \phi}(u) = \frac{1}{2} \left( \int_{\Sigma} |\nabla g u|^2 \, dv_g + \int_{\Sigma} \phi(u - \overline{\phi}) \, dv_g \right) - \rho \ln \int_{\Sigma} h e^u - \overline{\phi} \, dv_g,$$

where $h: \Sigma \to \mathbb{R}$ is a smooth positive function, $\phi: \mathbb{R} \to \mathbb{R}$ is a smooth one-variable function and $\overline{\phi} = \int_{\Sigma} \phi \, dv_g / |\Sigma|$. If $\rho \in (8k\pi, 8(k + 1)\pi)$ $(k \in \mathbb{N}^*)$, $\phi$ satisfies $|\phi(t)| \leq C ((|t|^p + 1)$ $(1 < p < 2)$ and $|\phi'(t)| \leq C (|t|^{p-1} + 1)$ for some constant $C$, then we get critical points of $J_{\rho, \phi}$ by adapting min-max schemes of Ding, Jost, Li and Wang [13], Djadli [14] and Malchiodi [22].

1. Introduction

Let $(\Sigma, g)$ be a closed Riemann surface and $\Delta_g$ be the Laplace–Beltrami operator. The mean field equation is known as

$$\Delta_g u = \rho \left( \frac{h e^u}{\int_{\Sigma} h e^u \, dv_g} - \frac{1}{|\Sigma|} \right).$$

2020 Mathematics Subject Classification. Primary: 49J35, 58J05; Secondary: 93B24.

Key words and phrases. Mean field equation; topological method; min-max scheme.

This research is partly supported by the National Natural Science Foundation of China (Grant No. 11721101), and by the National Key Research and Development Project SQ2020YFA070080.

The first author is supported by the Outstanding Innovative Talents Cultivation Funded Programs 2020 of Renmin University of China.