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CRITICAL POINTS OF A MEAN FIELD TYPE FUNCTIONAL ON A CLOSED RIEMANN SURFACE

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Abstract. Let (Σ, g) be a closed Riemann surface and $H^1(\Sigma)$ be the usual Sobolev space. For any real number ρ, we define a generalized mean field type functional $J_{\rho,\phi}\colon H^1(\Sigma) \to \mathbb{R}$ by

$$J_{\rho,\phi}(u) = \frac{1}{2}\bigg(\int_\Sigma |\nabla_g u|^2 \, dv_g + \int_\Sigma \phi(u - \overline{u}) \, dv_g\bigg) - \rho \ln \int_\Sigma h e^{u-\overline{u}} \, dv_g,$$

where $h\colon \Sigma \to \mathbb{R}$ is a smooth positive function, $\phi\colon \mathbb{R} \to \mathbb{R}$ is a smooth one-variable function and $\overline{u} = \int_\Sigma u \, dv_g / |\Sigma|$. If $\rho \in (8k\pi, 8(k+1)\pi)$ $(k \in \mathbb{N}^*)$, ϕ satisfies $|\phi(t)| \le C\,(|t|^p + 1)$ $(1 < p < 2)$ and $|\phi'(t)| \le C\,(|t|^{p-1} + 1)$ for some constant C, then we get critical points of $J_{\rho,\phi}$ by adapting min-max schemes of Ding, Jost, Li and Wang [13], Djadli [14] and Malchiodi [22].

1. Introduction

Let (Σ, g) be a closed Riemann surface and Δ_g be the Laplace–Beltrami operator. The mean field equation is known as

$$\Delta_g u = \rho\bigg(\frac{h e^u}{\int_\Sigma h e^u \, dv_g} - \frac{1}{|\Sigma|}\bigg), \tag{1.1}$$

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