

## GUTIERREZ–SOTOMAYOR FLOWS ON SINGULAR SURFACES

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*Dedicated to the memory of Professor Jorge Sotomayor*

ABSTRACT. In this work, we consider the collection of necessary homological conditions previously obtained via Conley index theory for a Lyapunov semi-graph to be associated to a Gutierrez–Sotomayor flow on an isolating block and address their sufficiency. These singular flows include regular  $\mathcal{R}$ , cone  $\mathcal{C}$ , Whitney  $\mathcal{W}$ , double  $\mathcal{D}$  and triple  $\mathcal{T}$  crossing singularities. Local sufficiency of these conditions are proved in the case of Lyapunov semi-graphs along with a complete characterization of the branched 1-manifolds that make up the boundary of the block. As a consequence, global sufficient conditions are determined for Lyapunov graphs labelled with  $\mathcal{R}$ ,  $\mathcal{C}$ ,  $\mathcal{W}$ ,  $\mathcal{D}$  and  $\mathcal{T}$  and with minimal weights to be associated to Gutierrez–Sotomayor flows on closed singular 2-manifolds. By removing the minimality condition, we prove other global realizability results by requiring that the Lyapunov graph be labelled with  $\mathcal{R}$ ,  $\mathcal{C}$  and  $\mathcal{W}$  singularities or that it be linear.

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