

**A PRIORI BOUNDS AND EXISTENCE  
OF POSITIVE SOLUTIONS  
FOR FRACTIONAL KIRCHHOFF EQUATIONS**

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ABSTRACT. In this paper, we are concerned with the following Kirchhoff equations involving the fractional Laplacian,

$$(0.1) \quad \begin{cases} (a + b[u]_s^2) (-\Delta)^s u = u^p + h(x, u, \nabla u), & x \in \Omega, \\ u > 0, & x \in \Omega, \\ u = 0, & x \notin \Omega, \end{cases}$$

where  $\Omega$  is a smooth bounded domain in  $\mathbb{R}^N$  ( $N \geq 3$ ),  $0 < s < 1$ ,  $a, b > 0$  and  $0 < p < (N + 2s)/(N - 2s)$  are constants. Under suitable conditions on  $h(x, u, \nabla u)$ , using the defining integral, we carry on a blowing-up and rescaling argument directly on the nonlocal equations and thus obtain a priori estimates on the positive solutions. Moreover, existence results for positive solutions of problem (0.1) are proved by Leray–Schauder degree theory and the above estimates.

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