A PRIORI BOUNDS AND EXISTENCE OF POSITIVE SOLUTIONS FOR FRACTIONAL KIRCHHOFF EQUATIONS

Pengfei Li — Junhui Xie — Dan Mu

Abstract. In this paper, we are concerned with the following Kirchhoff equations involving the fractional Laplacian,

\[
\begin{aligned}
(a + b|u|^2)^{s} u + h(x, u, \nabla u) &= u^p, & x \in \Omega, \\
 u &= 0, & x \notin \Omega,
\end{aligned}
\]

where \( \Omega \) is a smooth bounded domain in \( \mathbb{R}^N \) (\( N \geq 3 \)), \( 0 < s < 1 \), \( a, b > 0 \) and \( 0 < p < (N + 2s)/(N - 2s) \) are constants. Under suitable conditions on \( h(x, u, \nabla u) \), using the defining integral, we carry on a blowing-up and rescaling argument directly on the nonlocal equations and thus obtain a priori estimates on the positive solutions. Moreover, existence results for positive solutions of problem (0.1) are proved by Leray–Schauder degree theory and the above estimates.

2020 Mathematics Subject Classification. 35A16, 35B09, 35B44, 35B45, 35R11.

Key words and phrases. Fractional Kirchhoff equations; existence of solution; a priori bounds; Leray–Schauder degree theory.

The second author was supported in part by NSFC 11761030 and Cultivation Project for High-Level Scientific Research Achievements of Hubei Minzu University PY20002.