ON DETERMINING THE HOMOLOGICAL CONLEY INDEX OF POINCARÉ MAPS IN AUTONOMOUS SYSTEMS

ROMAN SRZEDNICKI

Dedicated to the memory of Professor Andrzej Granas

ABSTRACT. A theorem on computation of the homological Conley index of an isolated invariant set of the Poincaré map associated to a section in a rotating local dynamical system \( \phi \) is proved. Let \((N,L)\) be an index pair for a discretization \( \phi^h \) of \( \phi \), where \( h > 0 \), and let \( S \) denote the invariant part of \( N \setminus L \); it follows that the section \( S_0 \) of \( S \) is an isolated invariant set of the Poincaré map. The theorem asserts that if the sections \( N_0 \) of \( N \) and \( L_0 \) of \( L \) are ANRs, the homology classes \([u_j]\) of some cycles \( u_j \) form a basis of \( H(N_0,L_0) \), and for some scalars \( a_{ij} \), the cycles \( u_j \) and \( \sum a_{ij} u_i \) are homologous in the covering pair \((\tilde{N},\tilde{L})\) of \((N,L)\) and the homology relation is preserved in \((\tilde{N},\tilde{L})\) under the transformation induced by \( \phi^t \) for \( t \in [0,h] \) then the homological Conley index of \( S_0 \) is equal to the Leray reduction of the matrix \( [a_{ij}] \). In particular, no information on the values of the Poincaré map or its approximations is required. In a special case of the system generated by a \( T \)-periodic non-autonomous ordinary differential equation with rational \( T/h > 1 \), the theorem was proved in the paper M. Mrozek, R. Srzednicki, and F. Weilandt, SIAM J. Appl. Dyn. Syst. 14 (2015), 1348–1386, and it motivated a construction of an algorithm for determining the index.

2020 Mathematics Subject Classification. Primary: 37B30; Secondary: 37B35.

Key words and phrases. Poincaré map; isolated invariant set; index pair; Conley index.

This research is partially supported by the Polish National Science Center under Grant No. 2014/14/A/ST1/00453.