A WORD ON PROFESSOR KAZIMIERZ GOEBEL
(1940–2022)

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We lost an outstanding mathematician, Professor Kazimierz Goebel, our friend Kaz. He passed away on July 21, 2022. This is a tremendous loss for the mathematical community in general, but especially for our group in Lublin.

Kaz was born on October 21, 1940 in Warsaw, but soon after that his family moved to Lublin and this was the city in which he received his education and developed his academic career at the Maria Curie-Skłodowska University. At this university he started mathematical studies in 1958. One of his professors, Adam Bielecki recognized him as a very gifted student and directed into research problems. After graduation in 1963, Kaz got the position of teaching assistant at the university. He worked under supervision of professor Bielecki, but also attended the seminar of Jan Mikusiński at the Mathematical Institute of the Polish Academy of Science in Warsaw.

Kaz was granted Ph.D. degree in 1967 with the Ph.D. thesis *On lipschitzian mappings and their generalizations*. This was the beginning of his career. Fifty years later, in 2017, the ceremony of renewing his Ph.D. degree took place. That time he was recognized as an outstanding mathematician and had many scientific friends all over the word. He received several congratulation letters and a group of Spanish and Mexican mathematicians came to Lublin just to share the festivities with him. This was clear evidence how fruitful his scientific career was.

In the late sixties, inspired by the pioneering paper of William Art Kirk on existence of fixed points of nonexpansive mappings, Kaz started his research

on such mappings. He got the Kościuszko Foundation scholarship and visited Art in Iowa City. This was the turning point in his career. The academic year 1971/72 spent with Art was the beginning of the long-lasting collaboration and friendship. After over forty years, to emphasize how important this collaboration was for him and our Lublin research group, Kaz set forward an initiative to award professor Kirk the title of Honorary Doctor of UMCS. The ceremony of awarding the title took place in 2004, in presence of many guests from Poland and abroad.

Research in fixed point theory would not exist without the legacy of Stefan Banach, but it is not an exaggeration to say that Kirk and Goebel were the main founders of the modern metric fixed point theory. They wrote several fundamental joint papers on the subject, introducing new ideas and giving new directions of research. The most appreciated joint publication is their monograph *Topics in Metric Fixed Point Theory*, published by the Cambridge University Press in 1990. For many years it was the basic book for specialists working in the field and a source of inspiration for newcomers.

In 1971 Kaz received the habilitation qualification with the thesis *Thickness of sets in metric spaces and their applications to fixed point theory* and in 1977, at the age of 36, he was granted the title of extraordinary professor. Three years later he became the head of the Differential Equations Division in the Institute of Mathematics of UMCS.

We were lucky to have Kaz with us at UMCS. He inspired us as students with his brilliant lectures and next as colleagues during numerous discussions. He supervised 15 Ph.D. students and now three of them are full professors. For many years he led the seminar on fixed point theory. We had meetings with Kaz during the seminar time, but we also had long mathematical discussions in his office or just walking with him in the corridor. At the end of each academic year he organized for our group a barbecue in the university botanic garden to make a summary of the passing year. This was what he loved, meeting people and sharing the ideas. He used to say: I throw ideas and you can catch them.

He also loved traveling. He visited USA several times, usually passing through Iowa City to see Art. Every visit was fruitful. For instance, in 1981, Kaz was a visiting professor at the University of Southern California. During this visit he wrote with Simeon Reich the book *Uniform Convexity, Hyperbolic Geometry and Nonexpansive Mappings* published in 1983. It presents, typical for Kaz, geometric approach to fixed point problems, this time for holomorphic mappings on the open unit ball of a complex Hilbert space. Other favorite travel directions of Kaz included Italy, Spain, Australia, Thailand and Japan. He regularly attended conferences on nonlinear and convex analysis organized in Japan, was fond of Japanese culture and cuisine. In Spain his favorite destinations were Seville and Valencia. Colleagues from the universities in both cities have always emphasized

that he encouraged them to continue their research and offered many very useful advices. This was characteristic for Kaz. Being a strong person, he was also very friendly. Visiting different groups working on metric fixed point theory, he helped to consolidate them into one community. We were not only a group of experts working in the same field, but people who enjoyed being together. From 1989 to 2019, a series of conferences on metric fixed point theory and its applications were organized and Kaz was the only one who attended all of them. He was a member of organizing committees and a main speaker at most of them. Thanks to his initiative and efforts one of the conferences was organized in Kazimierz Dolny in 1997. The program of each conference included a party and Kaz was usually one of the leading figures. He proposed toasts, danced and sang songs. It seems that his favorite one was the most popular Warsaw city folklore song *Bal na Gnojnej*.

When Kaz turned seventy, special sessions of two conferences were dedicated to him: VIth *Symposium on Nonlinear Analysis* which was held in Toruń in 2011 and 10*th International Conference on Fixed Point Theory and Its Applications* which was held in Cluj-Napoca in 2012. Especially the first one was important for Kaz. He shared the special session with his friend Lech Górniewicz with whom he worked for many years in the Juliusz P. Schauder Center for Nonlinear Studies in Toruń.

Kaz wrote 75 research papers and the main areas of his research included: differential equations in Banach spaces, measures of noncompactness and their applications to fixed point theory and differential equations, fixed points of holomorphic mappings in Hilbert spaces, geometric properties of Banach spaces and their applications to fixed point theory, minimal displacement and optimal retraction problems. He also wrote five books. Two of them have been already mentioned. He wrote his first book *Measures of Noncompactness in Banach Spaces* with Józef Banaś in 1980. I had the privilege to be a coauthor of his last book *Elements of Geometry of Balls in Banach Spaces* which was published by the Oxford University Press in 2018. One of his books, *Concise Course on Fixed Point Theorems*, published in 2002, is based on a series of lectures given at Tokyo Institute of Technology. In MathSciNet it was reviewed in by Christian Fenske. He made the following comments, which give a very accurate characterization of the style of publications by Kaz.

"*The true value of the book, however, lies in the wealth of cleverly chosen examples and open problems. Only a true expert who has worked in the field of fixed point theory for more than three decades could have been able to present such a collection. As indicated by the title of the book, this is indeed a concise course, nevertheless it is a pleasure to read this text and everyone who has worked through the book will love the subject.*"

In 2011 Kaz was granted the award of the Polish Minister of Science and Higher Education for the lifetime scientific achievements. It is especially impressive that he managed to combine intensive and fruitful research with important duties in many scientific institutions and organizations, and the duties entrusted to him by the scientific community were really prominent. From 1993 to 1999 he was the rector of UMCS and the president of the Polish Mathematical Society. He was also a representative of Poland in the European Mathematical Society Council. For many years Kaz was a member of the Scientific Council of the Mathematical Institute of Polish Academy of Sciences. From 2005 to 2006 he was a member of the Presidium of the State Accreditation Committee. Working for this committee he visited many Polish universities to evaluate education standards and in cooperation with the Swiss counterpart of the committee he also paid such visit to the Federal Technical University in Lausanne.

Kaz dedicated himself to mathematics, but to make the picture more complete I should also mention that he loved sailing and swimming. He used to take part in the annual swimming relay-race organized by UMCS. In his recent races he was the oldest but not the slowest swimmer. Kaz had the yacht captain's certificate and in 1976 led the yacht Roztocze around Iceland. As in all areas of his activity, he played important role in organizing sailors community. He was the 18th member of the Polish branch (Mesa Kaprów Polskich) of Hermandad de la Costa (Brotherhood of the Coast), the society which cultivates sailors traditions, and from 1997 to 1999 he led the Mesa. Kaz had numerous collaborators and friends in Poland and in many places all over the world. I was one of them and I am sure that we will keep warm memories of him.