

**FIXED POINT THEOREMS
OF VARIOUS NONEXPANSIVE ACTIONS
OF SEMITOPOLOGICAL SEMIGROUPS
ON WEAKLY/WEAK* COMPACT CONVEX SETS**

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*In memory of Wataru Takahashi
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ABSTRACT. Let S be a right reversible semitopological semigroup, and let $\text{LUC}(S)$ be the space of left uniformly continuous functions on S . Suppose that $\text{LUC}(S)$ has a left invariant mean. Let K be a weakly compact convex subset of a Banach space not necessarily with normal structure. We show that there always exists a common fixed point for any jointly weakly continuous and super asymptotically nonexpansive action of S on K . Several variances involving the weak* compactness, the RNP, the distality of K and/or the left reversibility of S are also provided.

1. Introduction

Let K be a non-empty convex subset of a Banach space E . Let $T: K \rightarrow K$ be a *nonexpansive* map, namely $\|Tx - Ty\| \leq \|x - y\|$ for all x, y in K . Schauder [29] shows that T has a fixed point if K is norm compact. Kirk [13] shows that T has a fixed point if E is reflexive and K is weakly compact with normal structure.

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