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NONTRIVIAL SOLUTIONS FOR A CLASS OF GRADIENT-TYPE QUASILINEAR ELLIPTIC SYSTEMS

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Abstract. The aim of this paper is to investigate the existence of weak bounded solutions of the gradient–type quasilinear elliptic system

$$\text{(P)}\qquad \begin{cases} -\operatorname{div}(a_i(x,u_i,\nabla u_i)) + A_{i,t}(x,u_i,\nabla u_i) = G_i(x,\mathbf{u}) \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{in } \Omega \text{ for } i\in\{1,\dots,m\}, \\ \mathbf{u}=0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{on } \partial\Omega, \end{cases}$$

with $m \geq 2$ and $\mathbf{u} = (u_1,\dots,u_m)$, where $\Omega \subset \mathbb{R}^N$ is an open bounded domain and some functions $A_i\colon \Omega\times\mathbb{R}\times\mathbb{R}^N \to \mathbb{R}$, $i\in\{1,\dots,m\}$, and $G\colon \Omega\times\mathbb{R}^m \to \mathbb{R}$ exist such that $a_i(x,t,\xi) = \nabla_\xi A_i(x,t,\xi)$, $A_{i,t}(x,t,\xi) = \frac{\partial A_i}{\partial t}(x,t,\xi)$, and $G_i(x,\mathbf{u}) = \frac{\partial G}{\partial u_i}(x,\mathbf{u})$. We prove that, under suitable hypotheses, the functional $\mathcal{J}$ related to problem (P) is $\mathcal{C}^1$ on a "good" Banach space X and satisfies the weak Cerami–Palais–Smale condition. Then, generalized versions of the Mountain Pass Theorems allow us to prove the existence of at least one critical point and, if additionally $\mathcal{J}$ is even, of infinitely many critical points.

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Key words and phrases. Gradient-type quasilinear elliptic system; p-Laplacian type operator; subcritical growth; weak Cerami–Palais–Smale condition; Ambrosetti–Rabinowitz condition; Mountain Pass theorem; even functional; pseudo-eigenvalue.

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