

ON REEB GRAPHS INDUCED FROM SMOOTH FUNCTIONS ON 3-DIMENSIONAL CLOSED MANIFOLDS WITH FINITELY MANY SINGULAR VALUES

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ABSTRACT. The *Reeb graph* of a smooth function on a smooth manifold is the graph obtained as the space of all connected components of preimages (level sets) such that the set of all vertices coincides with the set of all the connected components of preimages containing some singular points. Reeb graphs are fundamental and important tools in algebraic topological and differential topological theory of Morse functions and their variants. In the present paper, as a related fundamental and important study, for given graphs, we construct certain smooth functions inducing the graphs as the Reeb graphs. Such results have been demonstrated by Masumoto, Michalak, Saeki, Sharko, among others, and also by the author since 2000s. We construct good smooth functions on suitable 3-dimensional connected, closed and orientable manifolds.

1. Introduction

First, throughout the present paper, graphs are fundamental tools. The *vertex set* of a graph is the set of all vertices and the *edge set* is the set of all edges. A graph is *finite* i.e. the vertex set and the edge set are finite and the

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