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ANALYTICAL AND COMPUTATIONAL RESULTS FOR THE DECAY OF SOLUTIONS OF A DAMPED WAVE EQUATION WITH VARIABLE-EXPONENT NONLINEARITIES

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Abstract. With the advancement of science and technology, many physical and engineering models require more sophisticated mathematical functional spaces to be studied and well understood. For example, in fluid dynamics, electrorheological fluids (smart fluids) have the property that the viscosity changes (often drastically) when exposed to an electrical field. The Lebesgue and Sobolev spaces with variable exponents proved to be efficient tools to study such problems as well as other models like the image processing. In this work, we consider the following nonlinear wave equation with variable exponents:

$$u_{tt} - \Delta u - \Delta u_t + |u_t|^{m(\,\cdot\,)-2}u_t = 0, \quad \text{in } \Omega \times (0,T),$$

where Ω is a bounded domain and $T > 0$, and show that weak solutions decay exponentially or polynomially depending on the range of the variable exponent m. We also give two numerical examples to illustrate our theoretical results.

1. Introduction

The stabilization of linear and nonlinear wave equations by means of internal or boundary feedbacks has attracted a considerable attention, and much effort

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