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SEMICLASSICAL STATES FOR A SCHRÖDINGER–POISSON SYSTEM WITH HARTREE-TYPE NONLINEARITY

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Abstract. In this paper we are interested in a class of semiclassical Schrödinger–Poisson systems with Hartree-type nonlinearites. Firstly, we prove the existence of groundstate for the autonomous system by using the subcritical approximation and the Pohozaev constraint method. Secondly, we prove the existence of semiclassical state solutions and multiplicity for the system with critical frequency by using the genus. Finally, we study multiplicity and concentration behavior of solutions of the system with general potential by using the Lusternik–Schnirelman theory.

1. Introduction

In this paper, we study the following Schrödinger–Poisson system:

$$\begin{cases} -\varepsilon^2\Delta u + V(x)u = \varepsilon^{\mu-3}(I_\mu * Q(y)|u|^{2_*})Q(x)|u|^{2_*-2}u \\ \qquad\qquad\qquad +Z(x)|u|^4u + K(x)\phi|u|^3u, & x \in \mathbb{R}^3, \\ -\varepsilon^2\Delta\phi = K(x)|u|^5, & x \in \mathbb{R}^3, \end{cases} \tag{1.1}$$

where $\varepsilon > 0$, V, Q, K, Z are real functions, $0 < \mu < 3$, and $2_* = (6-\mu)/3$ is the lower critical exponent in the sense of the Hardy–Littlewood–Sobolev inequality.

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