

EXISTENCE AND REGULARITY OF POSITIVE SOLUTIONS OF A DEGENERATE FOURTH ORDER ELLIPTIC PROBLEM

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ABSTRACT. In this paper, we consider existence and regularity of positive solutions of a degenerate fourth order elliptic problem. Firstly, a new Caffarelli–Kohn–Nirenberg type inequality for the fourth order case is established. Then, by the use of the corresponding embedding, we obtain the existence of positive solutions of a degenerate fourth order elliptic problem. Finally, the regularity of the positive solutions is also studied.

1. Introduction

In the paper, we consider the Navier boundary problem

$$(1.1) \quad \begin{cases} \Delta(a(x)\Delta u) = b(x)u^p & \text{in } \Omega, \\ u = \Delta u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 1$, and $p > 1$.

Let $\{z_1, \dots, z_k\} \subset \Omega$ and $a \in C^0(\overline{\Omega} \setminus \{z_1, \dots, z_k\})$, $b \in C^2(\overline{\Omega} \setminus \{z_1, \dots, z_k\})$ be non-negative functions satisfying $a(x) > 0$, $b(x) > 0$ for $x \in \overline{\Omega} \setminus \{z_1, \dots, z_k\}$

2020 *Mathematics Subject Classification*. Primary: 35J40; Secondary: 35B45.

Key words and phrases. Embedding theorem; fourth order operator equations; degenerate elliptic problem.

The first author was supported in part by NSF 11901158 and the Startup Foundation for Introducing Talent of NUIST.

The second author was supported in part by NSF 11571176.