

SOLUTIONS TO INDEFINITE WEAKLY COUPLED COOPERATIVE ELLIPTIC SYSTEMS

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To the memory of Andrzej Granas

ABSTRACT. We study the elliptic system

$$\begin{cases} -\Delta u_1 - \kappa_1 u_1 = \mu_1 |u_1|^{p-2} u_1 + \lambda \alpha |u_1|^{\alpha-2} |u_2|^\beta u_1, \\ -\Delta u_2 - \kappa_2 u_2 = \mu_2 |u_2|^{p-2} u_2 + \lambda \beta |u_1|^\alpha |u_2|^{\beta-2} u_2, \\ u_1, u_2 \in D_0^{1,2}(\Omega), \end{cases}$$

where Ω is a bounded domain in \mathbb{R}^N , $N \geq 3$, $\kappa_1, \kappa_2 \in \mathbb{R}$, $\mu_1, \mu_2, \lambda > 0$, $\alpha, \beta > 1$, and $\alpha + \beta = p \leq 2^* := 2N/(N-2)$. For $p \in (2, 2^*)$ we establish the existence of a ground state and of a prescribed number of fully nontrivial solutions to this system for λ sufficiently large. If $p = 2^*$ and $\kappa_1, \kappa_2 > 0$ we establish the existence of a ground state for λ sufficiently large if, either $N \geq 5$, or $N = 4$ and neither κ_1 nor κ_2 are Dirichlet eigenvalues of $-\Delta$ in Ω .

1. Introduction and statement of results

We consider the elliptic system

$$(1.1) \quad \begin{cases} -\Delta u_1 - \kappa_1 u_1 = \mu_1 |u_1|^{p-2} u_1 + \lambda \alpha |u_1|^{\alpha-2} |u_2|^\beta u_1, \\ -\Delta u_2 - \kappa_2 u_2 = \mu_2 |u_2|^{p-2} u_2 + \lambda \beta |u_1|^\alpha |u_2|^{\beta-2} u_2, \\ u_1, u_2 \in D_0^{1,2}(\Omega), \end{cases}$$

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