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BISECTION OF MEASURES ON SPHERES AND A FIXED POINT THEOREM

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Dedicated to the memory of Andrzej Granas

Abstract. We establish a variant for spheres of results obtained in [7], [3] for affine space. The principal result, that, if m is a power of 2 and $k \geq 1$, then km continuous densities on the unit sphere in $\mathbb{R}^{m+1}$ may be simultaneously bisected by a set of at most k hyperplanes through the origin, is essentially equivalent to the main theorem of Hubard and Karasev in [7]. But the methods used, involving Euler classes of vector bundles over symmetric powers of real projective spaces and an 'orbifold' fixed point theorem for involutions, are substantially different from those in [7], [3].

1. Introduction

Let V be a Euclidean vector space of dimension $m + 1$, $m > 0$. The unit sphere in V is denoted by $S(V)$ and the associated real projective space of 1-dimensional vector subspaces $L \subseteq V$ as $P(V)$.

A line $L \in P(V)$, with orthogonal complement $L^{\perp}$, determines an *equatorial slice* $S(L^{\perp})$ in $S(V)$ and this splits the sphere into two closed hemispheres parametrized by $e \in S(L)$:

$$S_e(V) = \{v \in S(V) \mid \langle v, e\rangle \geq 0\}$$

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