

## ASYMPTOTIC BEHAVIOR OF INEXACT ORBITS OF NONEXPANSIVE MAPPINGS

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**ABSTRACT.** We study the convergence of inexact iterates of nonexpansive mappings which take a nonempty closed subset of a complete metric space into the space in the case where the errors are sufficiently small.

### 1. Introduction

During more than fifty-five years now, there has been a lot of research activity regarding the fixed point theory of nonexpansive (that is, 1-Lipschitz) mappings. See, for example, [2]–[4], [7], [9]–[21], [24], [25] and the references cited therein. This activity stems from Banach’s classical theorem [1] concerning the existence of a unique fixed point for a strict contraction. It also covers the convergence of (inexact) iterates of a nonexpansive mapping to one of its fixed points. Since that seminal result, many developments have taken place in this field including, in particular, studies of feasibility and common fixed point problems, which find important applications in engineering and medical sciences [5], [6], [8], [22]–[25].

In [20] we collected several results which show the convergence of inexact orbits of a nonexpansive self-mapping of a complete metric space to one of its fixed points. In the present paper we establish three variants of these results for

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2020 *Mathematics Subject Classification.* 47H09, 47H10, 54E50.

*Key words and phrases.* Complete metric space; fixed point; inexact iteration; nonexpansive mapping.

The first author was partially supported by the Israel Science Foundation (Grant No. 820/17), by the Fund for the Promotion of Research at the Technion and by the Technion General Research Fund.