

**THE BROUWER DEGREE
ASSOCIATED TO CLASSICAL EIGENVALUE PROBLEMS
AND APPLICATIONS TO NONLINEAR SPECTRAL THEORY**

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*Dedicated to the memory of the outstanding mathematician Andrzej Granas,
whose contribution to nonlinear analysis has deeply inspired our research*

ABSTRACT. Thanks to a connection between two completely different topics, the classical eigenvalue problem in a finite dimensional real vector space and the Brouwer degree for maps between oriented differentiable real manifolds, we are able to solve, at least in the finite dimensional context, a conjecture regarding global continuation in nonlinear spectral theory that we formulated in some recent papers. The infinite dimensional case seems nontrivial, and is still unsolved.

1. Introduction

Consider the *nonlinear eigenvalue problem*

$$(1.1) \quad \begin{cases} L\mathbf{v} + sN(\mathbf{v}) = \lambda\mathbf{v}, \\ \mathbf{v} \in \mathbf{S}, \end{cases}$$

2020 *Mathematics Subject Classification.* 47J10, 47A75, 55M25.

Key words and phrases. Eigenvalues; eigenvectors; nonlinear spectral theory; degree theory.

A. Calamai is partially supported by G.N.A.M.P.A. – INdAM (Italy).

The first, second and fourth authors are members of the Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni (GNAMPA) of the Istituto Nazionale di Alta Matematica (INdAM).