Topological Methods in Nonlinear Analysis
Volume 59, No. 2A, 2022, 467–474
DOI: 10.12775/TMNA.2020.025

ON THE NUMBER OF STABLE POSITIVE SOLUTIONS OF WEAKLY NONLINEAR ELLIPTIC EQUATIONS WHEN THE DIFFUSION IS SMALL

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Abstract. We study the exact number of stable positive solutions of weakly nonlinear elliptic equations with small diffusion under rather general conditions on the nonlinearity.

In this paper, we continue our work on the number of positive stable solutions of problems

$$(1)\qquad \begin{aligned} -\varepsilon^2\Delta u &= f(u) && \text{in } \Omega,\\ u &> 0 && \text{in } \Omega,\\ u &= 0 && \text{on } \partial\Omega, \end{aligned}$$

where f is C^1, $f(0) \geq 0$, ε is small and positive, and Ω is a smooth bounded domain in $\mathbb{R}^3$. Here a solution is u on Ω is defined to be stable if

$$\int_\Omega \bigg(\frac{1}{2}\,\varepsilon(\nabla v)^2 - f'(u)v^2\bigg) \geq 0$$

for all smooth $v \in C_c^\infty(\Omega)$. In fact since our stable solutions on Ω are constructed by sub- and supersolutions, if Ω is bounded this turns out to be equivalent to

2020 *Mathematics Subject Classification.* 35J61, 35B35, 35J70, 35B44.
Key words and phrases. Small diffusion; stability of solutions to elliptic equations; blow-up.
This work was partially supported by the Australian Research Council.