

ANDRZEJ GRANAS
(1929–2019)

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A few years ago, Andrzej said to me: “We lived in interesting times, you know”. Coming from Andrzej, who was born on April 4, 1929 in Łódź, these words were quite fitting. Romana Granas, Andrzej’s mother, recalled once that her mother “saved my Andrzej”. This remark referred to events during their flight from the German army, which in 1941 had just invaded the part of Poland that had been occupied since 1939 by the Soviet Union.

According to family recollection, after 1941 Andrzej went to school in Tashkent. Andrzej’s mother told me how one day, back in Poland – I think it was in 1947 – Andrzej told her that he had decided to enroll at the university. When she asked him if he realized he needed a high school diploma in order to do so, he replied, “I’ve already passed my high school exams”. In the papers Andrzej left behind, there is a secondary school graduation certificate from the Kościuszko High School in Łódź, dated June 28, 1947. I remember that in the first post-war years, middle and high school classes included a special accelerated track which allowed older students to complete high school. I presume that Andrzej was able in this manner to make up for the time he had lost during the war.

However, obtaining his graduation certificate was not made any easier by the fact that his mother had given him a piano for his seventeenth birthday. This

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opened up an entirely new world for him and he began to study the piano with such zeal that – according to a couple of his old school friends – he didn't show up in class for months on end. In time, he came to play the instrument quite well, despite beginning so late.

He studied mathematics at the University of Warsaw from 1947 to 1952. At that time, and despite the losses sustained during the war, Poland was a powerhouse of mathematics. I can't remember where I read that when Stanisław Mikołajczyk was a deputy prime minister, he said that Poland could export two things: coal and mathematics. On many occasions, Andrzej recalled how the lectures of Kazimierz Kuratowski, Stanisław Mazur, and Karol Borsuk had an enormous impact on his mathematical development. Under Borsuk's influence, he became interested in the theory of retracts and in homotopy theory, particularly in cohomotopy groups.

From 1952 to 1955, Andrzej was a doctoral student (*aspirant*) at the University of Moscow, where he obtained his doctorate (*kandidat nauk*) in 1958. His supervisors were Lazar Aronovich Lusternik and Borsuk. He wrote his dissertation on applications of the antipodes theorem to functional spaces, in reference to the pre-war work of Jean Leray and Juliusz Schauder, who had pioneered research on the application of topology to nonlinear problems of functional analysis. Andrzej's mathematical horizons broadened considerably thanks to his contacts with Lusternik and the opportunities inherent to studying in Moscow, which was a rapidly growing center of algebraic topology – a modern current in contemporary mathematics at the time.

From 1955 to 1959, Andrzej worked at the Nicolaus Copernicus University (UMK) in Toruń and maintained close ties with the Mathematical Institute of the Polish Academy of Sciences (IM PAN) in Warsaw, by participating in, among other activities, the seminars conducted by Borsuk and Kuratowski. In his article 50 Years of the Algebraic Seminar in Toruń, Stanisław Balcerzyk recalls that:

Prof. Andrzej Granas, who after completing his aspirant program in Moscow worked at UMK from 1956 to 1959, also had an important influence on the course of the seminar. In Moscow he had come into close contact with algebraic topology and sought to convey to us the essential elements of this theory. Grasping it proved difficult, and the choice as reading of one of the chapters from P.S. Aleksandrov's *Kombinatornaya topologia* was definitely unsuccessful. Only the publication in 1958 of the Russian translation of Eilenberg–Steenrod's book, and later of S. Hu's book, made it possible to learn the fundamentals of these theories. [...] The topological and algebraic preparation thus obtained enabled E. Sasiada and myself to begin studying homological algebra.

From this first Toruń period I remember very well how Andrzej brought from Warsaw several copies of the Russian translation of Foundations of Algebraic Topology by Samuel Eilenberg and Norman Steenrod and organized something like a study group, in which specific chapters of this work were discussed. Among those who took an active part in these meetings were Stanisław Balcerzyk, Edward Szaśiada, Andrzej Jankowski (still a third-year student at the time), and myself. For me, participating in these meetings was very valuable, as they not only allowed me to learn the basics of algebraic topology but also gave me an opportunity to become familiar with the methods of category theory. It should be added that due to this collaboration, Stanisław Balcerzyk wrote a book and Edward Szaśiada wrote a paper.

In 1958–1960, the game of bridge was played quite often in Toruń. It was well known that, when doubled, Andrzej would, as if out of principle, redouble. I remember perfectly the following scene: Andrzej was playing with Leon Jeśmanowicz as partner and they were bidding a very high contract. One of the opponents doubled and Andrzej, of course, redoubled and was the declarer. Jeśmanowicz had graduated from Stefan Batory University in Wilno (now Vilnius in Lithuania), and was a highly cultured man of great knowledge and education who spoke perfect pre-war Polish, but when he wished, he could speak with the Eastern Borderlands dialect. After the bidding ended, Jeśmanowicz laid out his cards on the table, got up and walked around the table to look at Andrzej's hand and said, with a pronounced Vilnius accent especially reserved for such occasions: "Well, them cards cry belly-up to me".

In 1960–1961 Andrzej was in the United States at the invitation of the University of Chicago and the Institute of Advanced Study in Princeton. After his return to Poland, he worked for IM PAN, where he obtained his habilitation in 1961 by presenting a dissertation which followed-up on the inter-war work of Jean Leray and Juliusz Schauder on applications of topology in analysis, and initiated the revival of this subject in Polish post-war mathematics.

In 1962–1965 Andrzej was a visiting professor for two years at the University of Georgia in Athens and for one year at McGill University in Montreal.

In 1965–1969 he worked at Gdańsk College of Education (WSP) – in addition to IM PAN – and was director of the chair of topology, which had been especially established for him. In 1967 he obtained the title of professor.

I still remember an event that illustrates the great contrast between Andrzej's approach to playing bridge and to doing mathematics. In order to prove a certain lemma, I had created a diagram that was so complicated that it could hardly be contained on an A4 sheet of paper. To make it more legible, I redrew it on an A3-size sheet and handed it to Andrzej. Over the next few days he praised the diagram and its dimensions. A little while later he wrote an elegant proof of

the same lemma, in which – by introducing some natural definitions of objects appearing in the diagram – he reduced the entire proof to a few sentences, so that my diagram became irrelevant. This proof, as a proof of Theorem 5.2, is given in Chapter 6 of [7].

On the occasion of the International Congress of Mathematics in Warsaw in 1983 (the congress had been planned for 1982 and was postponed for a year on account of martial law) Springer publishing house had a poster printed concerning the most eminent Polish mathematicians. I remember how Andrzej worked for two weeks, reverently and with painstaking attention to detail, on the draft version of this poster.

In 1965–1969, Andrzej took advantage of the opportunities created by his simultaneous employment at WSP in Gdańsk and at IM PAN, and established a branch of the latter institute in Sopot. Andrzej's seminar was an important part of the branch's activities. It was regularly attended by a dozen or so young mathematicians from the Baltic Coast area as well as speakers and invited guests from Poland and abroad. I am unable to provide an exhaustive list of the guests who visited the branch during that period, but I should mention that they included mathematicians of such standing as René Thom and Peter Hilton.

Andrzej was a great admirer of the Polish mathematical school of the interwar period. When I think of his observations on the achievements of Polish mathematicians of that day, they bring to mind a football fan speaking of the successes of the Polish national team in the days when Kazimierz Górski was its head coach. At the same time, thanks to his sojourns in Moscow and the United States, he was fully aware that the end of the fifties and beginning of the sixties was a period of great achievements in algebraic and differential topology, and he deplored the fact that these results attracted so little attention in Poland. As a first step toward changing this state of affairs, he organized a conference in Toruń in 1962 devoted to the basic concepts of Leray's theory of spectral sequences.

In 1967 Andrzej organized the first Summer School on Algebraic Topology in Gdańsk. Its main lecturers were Stanisław Balcerzyk and Andrzej Białynicki-Birula. This school was the first of an entire series of annual summer schools held in Gdańsk and Sopot. After Andrzej left Poland in 1970, the organization of these schools was taken over – in various configurations – by Bogdan Bojarski, Andrzej Jankowski, Stefan Jackowski, and myself, with the indefatigable participation of Zygfryd Kucharski, who took care of the thankless technical and organizational aspects.

In 1970 Andrzej left Poland, and he remained abroad for over twenty years. During the first half of 1970, he was in Paris at the invitation of the Collège de France. From the fall of 1970 until his retirement in 1991 he was a professor

at the Université de Montréal in Canada. His period in Montreal was a highly productive time. Taking advantage of the opportunities provided by his position as a professor at this Canadian university, he continued his organizational activities with great energy and persistence. The Université de Montréal – with NATO support – organized an annual conference called the Séminaire de Mathématiques Supérieures. Andrzej was the main organizer of those conferences in 1973, 1983, 1986, and 1994. An additional circumstance favoring visits by Polish mathematicians to Montreal was the fact that LOT Polish Airlines flights between Warsaw and the United States made stopovers in Montreal.

Following his retirement, from 1991 to 1996 Andrzej divided his time between Montreal and Toruń, as professor emeritus at the Université de Montréal and professor at the Nicolaus Copernicus University in Toruń. In 1991, he initiated the creation in Toruń of the Juliusz P. Schauder Center for Nonlinear Studies and became its first director. Through his efforts, in 1993, the Polish Center for Scientific Research, in cooperation with the American Mathematical Society, began publishing *Topological Methods in Nonlinear Analysis*, a journal affiliated with the Schauder Center. Andrzej was editor-in-chief of the journal during this period, and through his work the title gained international recognition.

Andrzej was also the initiator and for several years editor-in-chief of the Birkhäuser-published journal *Fixed Point Theory and Applications*, the first volume of which appeared in 2007. Andrzej's academic output includes two books on fixed point theory and ninety papers. The first of these books, published in 1982, had "Vol. I" in the title, and a second volume was planned. James Dugundji's severe illness and subsequent death in 1985 shattered these plans. Andrzej published a second book over twenty years after the first, with Dugundji listed as a co-author. However, this book was not the planned second volume but a comprehensive monograph incorporating the material contained in the first. After publication of the second book, questions naturally arose about Dugundji's contribution. I do not know what material traces of Dugundji's work on the second volume remain. I do know, however, that both authors put a great deal of effort into its preparation, that they had worked on its general premises since 1978, and that they discussed some of the issues it contains in great detail.

I traveled to Montreal several times on Andrzej's invitation in the 1980s and I remember very clearly many details of a long discussion I had there with Dugundji in 1983 – after the first volume had appeared – and which concerned the method of proof of Borsuk's theorem on antipodes. When I read the proof of the theorem on antipodes in the second volume in 2003, I remembered this discussion. In his extensive review published in the *AMS Bulletin*, Robert Brown characterized the book brilliantly by writing that "...the interaction between topology and nonlinear analysis is a persistent theme of the book..." and "[t]he

reader may be surprised by the relative brevity of most of the proofs. This is the consequence of the authors' very efficient style: we are told just the essential steps that make the statement of the result believable". In the preface to the second book, Andrzej wrote that Dugundji was "... totally independent and would not tolerate anything which he considered second best". I think these words applied to both authors.

After his arrival in Montreal, Andrzej published several dozen papers. In a preprint which appeared in Taiwan, he divided them as follows:

- **Topology of functional spaces.** In this group we find ten papers thematically related to Andrzej's habilitation dissertation and nine papers on the generalization of cohomology theory to a category whose objects are closed and bounded subsets of a fixed Banach space and whose morphisms are compact disorders of identities. The results of these nine papers are collected in [6].
- **Fixed Point Theory.** A group of eighteen papers, devoted to various variants of fixed point theory. This work is well represented by an article which contains a positive answer to Borsuk's question concerning fixed points of mappings on the approximate ANR.
- **Nonlinear Equations.** A group of twenty four papers on topological theorems having applications in the theory of differential equations. A typical representative is the paper devoted to the proof of an infinite-dimensional version of the classical theorem of homotopy theory.
- **Convex Analysis.** A series of nine papers on a special class of convex mappings. The work on these mappings is inspired by Problem 54 of the Scottish Book, formulated by Schauder.
- **Ordinary Differential Equations.** A series of twenty papers on applications of topological methods to the theory of ordinary differential equations. The subject in two of these papers is applications of the "topological transversality" method to differential equations.

When discussing the academic output of mathematicians, their lectures and readings must not be omitted. In this respect Andrzej's achievements are truly impressive. According to a list ending in 1978, he had lectured at thirty American and European institutions, including the universities of Bonn, Florence, Paris-Orsay, and California. After 1978, he lectured at various academic institutions in Europe and North America, as well as in Australia, Japan, Taiwan, China, and Israel.

Andrzej passed away on March 5, 2019. He is buried in Powązki Communal Cemetery in Warsaw. He left behind two books and ninety published papers. He founded two journals, organized the Juliusz P. Schauder Center of Nonlinear Studies in Toruń, and supervised sixteen PhDs in the mathematical sciences.

Andrzej was the hero of many anecdotes – some real, some embellished, some entirely fabricated – that made the rounds among mathematicians. The much regretted Andrzej Lasota formulated the rule that “in Granas’s neighbourhood, the density of probability increases”.

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