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ON GLOBAL BIFURCATION FOR THE NONLINEAR STEKLOV PROBLEMS

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Abstract. For $p \in (1,\infty)$, for an integer $N \geq 2$ and for a bounded Lipschitz domain $\Omega \subset \mathbb{R}^N$, we consider the following nonlinear Steklov bifurcation problem

$$-\Delta_p \phi = 0 \quad \text{in } \Omega, \qquad |\nabla \phi|^{p-2}\frac{\partial \phi}{\partial \nu} = \lambda\big(g|\phi|^{p-2}\phi + f r(\phi)\big) \quad \text{on } \partial\Omega,$$

where Δ_p is the p-Laplace operator, $g, f \in L^1(\partial\Omega)$ are indefinite weight functions and $r \in C(\mathbb{R})$ satisfies $r(0) = 0$ and certain growth conditions near zero and at infinity. For f, g in some appropriate Lorentz–Zygmund spaces, we establish the existence of a continuum that bifurcates from $(\lambda_1, 0)$, where λ_1 is the first eigenvalue of the following nonlinear Steklov eigenvalue problem

$$-\Delta_p \phi = 0 \quad \text{in } \Omega, \qquad |\nabla \phi|^{p-2}\frac{\partial \phi}{\partial \nu} = \lambda g|\phi|^{p-2}\phi \quad \text{on } \partial\Omega.$$

1. Introduction

Let Ω be an open bounded Lipschitz domain in $\mathbb{R}^N$ $(N \geq 2)$ with the boundary $\partial\Omega$. For $p \in (1,\infty)$, we consider the following nonlinear Steklov bifurcation

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