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PROPER k-BALL-CONTRACTIVE MAPPINGS IN $C_b^m[0,+\infty)$

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Abstract. In this paper we deal with the Banach space $C_b^m[0,+\infty)$ of all m-times continuously derivable, bounded with all derivatives up to the order m, real functions defined on $[0,+\infty)$. We prove, for any $\varepsilon > 0$, the existence of a new proper k-ball-contractive retraction with $k < 1+\varepsilon$ of the closed unit ball of the space onto its boundary, so that the Wośko constant $W_\gamma(C_b^m[0,+\infty))$ is equal to 1.

1. Introduction

Given a Banach space X, we denote by $B(X) = \{x \in X : \|x\| \le 1\}$ the closed unit ball and by $S(X) = \{x \in X : \|x\| = 1\}$ the unit sphere in X. It is well known that in any infinite-dimensional Banach space X there is a retraction from $B(X)$ onto $S(X)$, that is, a continuous mapping $R\colon B(X) \to S(X)$ such that $Rx = x$ for $x \in S(X)$. Moreover, such a retraction can be chosen to be Lipschitzian [5] with $\|Rx - Ry\| \le k_0\|x - y\|$, for some universal constant k_0. The optimal retraction problem, considered for the first time in [20], consists in the evaluation, in a given Banach space X, of the constant $k_0(X)$ which is the infimum of all k for which there exists a retraction of $B(X)$ onto $S(X)$ being Lipschitz with constant k. The problem has found a large interest in the literature. It is known $k_0(X) \ge 3$ for every space X. For the evaluation of the

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