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PERIODIC SOLUTIONS FOR SYSTEMS OF FUNCTIONAL-DIFFERENTIAL SEMILINEAR EQUATIONS AT RESONANCE

Pablo Amster — Julián Epstein — Arturo Sanjuán Cuéllar

Abstract. Motivated by Lazer–Leach type results, we study the existence of periodic solutions for systems of functional-differential equations at resonance with an arbitrary even-dimensional kernel and linear deviating terms involving a general delay of the form $\int_0^{2\pi} u(t+s)\,d\lambda(s)$, where λ is a finite regular signed measure. Our main technique shall be the Coincidence Degree Theorem due to Mawhin.

1. Introduction

The problem of finding periodic solutions to semilinear functional-differential equations at resonance has been widely studied. For instance, in the simplest situation of a scalar ordinary differential equation, nonlinear perturbations of harmonic oscillators of the form

$$x'' + m^2 x + g(x) = p(t) \tag{1.1}$$

were considered by Lazer and Leach among others, see [4] for $m = 0$ and [5] for strictly positive integer values of m. Equations like (1.1) are also known as Duffing equations at resonance. In [5], periodic solutions to (1.1) were found

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