NONLOCAL ELLIPTIC EQUATION
WITH CRITICAL EXPONENTIAL GROWTH
AND RESONANCE IN HIGH-ORDER EIGENVALUES

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Abstract. In this paper we are interested in the following nonlocal elliptic
equation

$$\begin{cases} -\Delta u = \lambda_k u + \left[\dfrac{1}{|x|^\mu} * G(x,u)\right] g(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where an open $\Omega \subset \mathbb{R}^2$ is bounded with smooth boundary. The nonlinearity $g(x,s)$ has the critical exponential growth in the sense of the Trudinger–Moser inequality and λ_k denotes the kth eigenvalue of $(-\Delta, H_0^1(\Omega))$, $k \geq 2$. Employing variational methods we prove the existence of a nontrivial solution for this nonlocal elliptic problem.

1. Introduction and main results

For the past few years, many researchers focus on the following nonlocal elliptic equation

$$(1.1) \qquad -\Delta u + V(x)u = \left[\frac{1}{|x|^\mu} * F(u)\right]f(u) \quad \text{in } \mathbb{R}^N,$$