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# NODAL SOLUTIONS FOR A CRITICAL KIRCHHOFF TYPE PROBLEM IN $\mathbb{R}^N$

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Abstract. In the present paper, we concentrate on the following critical Kirchhoff type problem

$$-\left(a+b\int_{\mathbb{R}^N}|\nabla u|^2\,dx\right)\triangle u+u=|u|^{2^*-2}u+\mu|u|^{p-2}u,\quad \text{in } \mathbb{R}^N,$$

where $N\geq 3$, $a,b>0$, $p\in(2,\ 2^*)$ and $\mu$ is an arbitrary positive parameter. With the help of an equivalent transformation, we first obtain at least one ground state nodal solution with precisely two nodal domains for $N=3$, all $b>0$ and $N\geq 4$, $b>0$ small enough. Moreover, we give a convergence property of ground state nodal solutions as $b\searrow 0$. Besides, we attain infinitely many nodal solutions for $N=3$, $p\in(4,6)$, all $b>0$ and $N\geq 4$, $p\in(2,\ 2^*)$, $b>0$ sufficiently small, and also establish nonexistence results of nodal solutions for $N\geq 4$ and $b$ large enough.

## 1. Introduction

In this paper, we investigate the existence and asymptotic behavior of ground state nodal solutions, the multiplicity and nonexistence of nodal solutions for the following Kirchhoff type problem

$$-\left(a+b\int_{\mathbb{R}^N}|\nabla u|^2\,dx\right)\triangle u+u=|u|^{2^*-2}u+\mu|u|^{p-2}u,\quad \text{in } \mathbb{R}^N, \tag{1.1}$$

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