

ASYMPTOTIC AUTONOMY OF BI-SPATIAL ATTRACTORS FOR STOCHASTIC RETARDED NAVIER–STOKES EQUATIONS

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ABSTRACT. We establish semi-convergence of a non-autonomous bi-spatial random attractor towards to an autonomous attractor under the topology of the regular space when time-parameter goes to infinity, where the criteria are given by forward compactness of the attractor in the terminal space as well as forward convergence of the random dynamical system in the initial space. We then apply to both non-autonomous and autonomous stochastic 2D Navier–Stokes equations with general delays (including variable and distribution delays). The forward-pullback asymptotic compactness in the space of continuous Sobolev-valued functions is proved by the method of spectrum decomposition.

1. Introduction

The asymptotically autonomous problem of a pullback attractor seems to have been investigated by Koleden and Simsen [21] (or see [7]). The criteria have been relaxed or improved in [11], [12], [22], [30], [36] and generalized to the stochastic case in [42], where the asymptotic semi-convergence of the attractor holds with respect to the topology of the initial Banach space X only.

Unlike the articles mentioned above, we study the asymptotic semi-convergence of a *bi-spatial random attractor* in the terminal space Y , which often has a stronger topology than X . More precisely, let $\mathcal{A} = \{\mathcal{A}(\tau, \omega)\}$ (resp. $\mathcal{A}_\infty =$

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