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FINITENESS IN POLYGONAL BILLIARDS ON HYPERBOLIC PLANE

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Abstract. J. Hadamard studied the geometric properties of geodesic flows on surfaces of negative curvature, thus initiating "Symbolic Dynamics". In this article, we follow the same geometric approach to study the geodesic trajectories of billiards in "rational polygons" on the hyperbolic plane. We particularly show that the billiard dynamics resulting thus are just 'Subshifts of Finite Type' or their dense subsets. We further show that 'Subshifts of Finite Type' play a central role in subshift dynamics and while discussing the topological structure of the space of all subshifts, we demonstrate that they approximate any shift dynamics.

1. Introduction

'Mathematical billiards' describe the motion of a point mass in a domain with elastic reflections from the boundary, and occur naturally in many problems in science. The billiards problem has typically been studied in planar domains.

A *billiard* in a 'domain' Π in the Euclidean plane is defined as a dynamical system described by the motion of a point-particle within Π along the straight lines with specular reflections from the boundary $\partial\Pi$. A domain is generally taken to be a subset of the plane that is compact with a piecewise smooth boundary. We refer to the point-particle under consideration as the *billiard ball*, the path followed within Π as the *billiard trajectory* and the respective domain is called the *billiard table*. This simple to describe mathematical system captures

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