Topological **M**ethods in **N**onlinear **A**nalysis
Volume 58, No. 2, 2021, 453–480
DOI: 10.12775/TMNA.2020.075

SEPARATING SOLUTIONS OF NONLINEAR PROBLEMS USING NONLINEAR GENERALIZED RAYLEIGH QUOTIENTS

Marcos Leandro Carvalho — Yavdat Il'yasov
Carlos Alberto Santos

Abstract. This paper deals with nonlinear elliptic boundary value problems with complicated geometry of nonlinearities. A new method for obtaining multiple solutions based on a recursive use of the nonlinear generalized Rayleigh quotients to the split Nehari manifold into subsets without degeneracies is introduced. The method is applied to prove the multiplicity result for nonnegative solutions, as well as to find a ground state of elliptic boundary value problems with nonlinearities of polynomial type.

1. Introduction

We consider the parametrized boundary value problem

$$\text{(P)} \qquad \begin{cases} -\Delta u = f_{\overline{\lambda}}(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 1$, is a smooth bounded domain, $\overline{\lambda} \in \mathbb{R}^m$ with $m \geq 1$ is a vector-parameter and $f_{\overline{\lambda}}\colon \Omega \times \mathbb{R} \to \mathbb{R}$. This equation has a variational form

2020 *Mathematics Subject Classification.* Primary: 35J60, 35J20, 35B30; Secondary: 35J05, 35J15.

Key words and phrases. Nonlinear generalized Rayleigh quotient; multiplicity result; Nehari manifold; ground state solution.

The last author was supported in part by CAPES/Brazil Proc. N^o 2788/2015-02.