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THE BORSUK–ULAM PROPERTY FOR MAPS FROM THE PRODUCT OF TWO SURFACES INTO A SURFACE

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Abstract. Let X, Y, S be closed connected surfaces and $\tau \times \beta$ a diagonal involution on $X \times Y$ where τ and β are free involutions on X and Y, respectively. In this work we study when the triple $(X\times Y, \tau\times\beta, S)$ satisfies the *Borsuk–Ulam property*. The problem is formulated in terms of an algebraic diagram, involving the 2-string braid group $B_2(S)$.

1. Introduction

The classical Borsuk–Ulam theorem as proved in [1] has been extensively generalized and its generalizations studied along the past 70 years and more recently have been the subject of many works. To see some of the recent developments see [4] and references there in, without certainly exhausting all the relevant works on the subject. We recall a version of a such generalization. Let V, Z be two topological spaces and α a fixed point free involution on V. The triple $(V, \alpha; Z)$ is said to satisfy the *Borsuk–Ulam property* if for every continuous map $f\colon V \to Z$ there exists a point $v \in V$ such that $f(\alpha(v)) = f(v)$.

Let X, Y, S be closed connected surfaces and denote by $\tau \times \beta$ the diagonal involution on $X\times Y$ where τ and β are free involutions on X and Y, respectively. The main purpose of this work is to answer when does the triple $(X\times Y, \tau\times\beta; S)$ satisfy the *Borsuk–Ulam property*? This question has been solved in [6] when S

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