

SOME BIFURCATION RESULTS AND MULTIPLE SOLUTIONS FOR THE p -LAPLACIAN EQUATION

MINGZHENG SUN — JIABAO SU — LEIGA ZHAO

ABSTRACT. In this paper, we first give some bifurcation results near the origin for the p -Laplacian equation, then multiple solutions are obtained based on the combination of perturbation methods in critical groups and minimax methods.

1. Introduction

Let Ω be a bounded domain in \mathbb{R}^N with smooth boundary $\partial\Omega$, and we consider the following quasilinear elliptic equation

$$(P) \quad \begin{cases} -\Delta_p u = \lambda|u|^{p-2}u + f(x, u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ with $1 < p < \infty$, and $\lambda \in \mathbb{R}$ is a parameter. We make the following assumptions on f :

(f₁) $f \in C(\Omega \times \mathbb{R}, \mathbb{R})$, $f(x, 0) = 0$ and

$$\lim_{|t| \rightarrow 0} \frac{f(x, t)}{|t|^{p-2}t} = 0, \quad \text{for } x \in \Omega;$$

2020 *Mathematics Subject Classification*. Primary: 35B32, 58E05; Secondary: 35J92.

Key words and phrases. Morse theory; p -Laplacian; bifurcation; multiple solutions.

The paper was supported by the KZ202010028048, KM201710009012, NSFC (11771302, 12171326, 12171014), NCUT (110052971921/102, 110052972027/014) and BTBU (19008021182).