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EXTENSION OF LIPSCHITZ-TYPE OPERATORS ON BANACH FUNCTION SPACES

Wasthenny V. Cavalcante — Pilar Rueda
Enrique A. Sánchez-Pérez

Abstract. We study extension theorems for Lipschitz-type operators acting on metric spaces and with values on spaces of integrable functions. Pointwise domination is not a natural feature of such spaces, and so almost everywhere inequalities and other measure-theoretic notions are introduced. We analyze Lipschitz-type inequalities in two fundamental cases. The first concerns almost everywhere pointwise inequalities, while the second considers dominations involving integrals. These Lipschitz-type inequalities provide a suitable frame to work with operators that take values on Banach function spaces. In the last part of the paper we use some interpolation procedures to extend our study to interpolated Banach function spaces.

1. Introduction and basic definitions

Extension of Lipschitz functions acting on subsets of metric spaces is a relevant issue in mathematical analysis, not only because of its theoretical interest but also because of the large number of applications that have been obtained. There are two classical extension results that are considered as milestones in the theory. The first one is the McShane–Whitney theorem, which concerns real

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