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QUASILINEAR SCHRÖDINGER EQUATIONS WITH SINGULAR AND VANISHING POTENTIALS INVOLVING NONLINEARITIES WITH CRITICAL EXPONENTIAL GROWTH

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Abstract. In this paper, we study the following class of Schrödinger equations:

$$-\Delta_N u + V(|x|)|u|^{N-2}u = Q(|x|)h(u) \quad \text{in } \mathbb{R}^N,$$

where $N \geq 2$, $V, Q\colon \mathbb{R}^N \to \mathbb{R}$ are potentials that can be unbounded, decaying or vanishing at infinity and the nonlinearity $h\colon \mathbb{R} \to \mathbb{R}$ has a critical exponential growth concerning the Trudinger–Moser inequality. By using a variational approach, a version of the Trudinger–Moser inequality and a symmetric criticality type result, we obtain the existence of nonnegative weak and ground state solutions for this class of problems and under suitable assumptions, we obtain a nonexistence result.

1. Introduction

In this paper, we deal with the existence of nonnegative weak solution for the following class of Schrödinger equations:

$$(\mathcal{P}) \qquad \begin{cases} -\Delta_N u + V(|x|)|u|^{N-2}u = Q(|x|)h(u) & \text{for } x \in \mathbb{R}^N, \\ u(x) \to 0 & \text{if } |x| \to +\infty, \end{cases}$$

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