Topological **M**ethods in **N**onlinear **A**nalysis
Volume 57, No. 1, 2021, 275–295
DOI: 10.12775/TMNA.2020.030

FRACTIONAL KIRCHHOFF–SCHRÖDINGER EQUATION WITH CRITICAL EXPONENTIAL GROWTH IN $\mathbb{R}^N$

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Abstract. In this paper, we consider the following fractional Kirchhoff–Schrödinger equation:

$$\begin{cases} \Big(a+b\|u\|_E^{p(\theta-1)}\Big)\big[(-\triangle)_p^s u+V(x)|u|^{p-2}u\big]=f(x,u), & x\in\mathbb{R}^N, \\ \|u\|_E^p:=\displaystyle\iint_{\mathbb{R}^{2N}}\frac{|u(x)-u(y)|^p}{|x-y|^{N+sp}}\,dx\,dy+\int_{\mathbb{R}^N}V(x)|u|^p\,dx, \end{cases}$$

where $a>0$, $b\geq 0$, $\theta\geq 1$, dimension $N=sp$ with $s\in(0,1)$ and $p\geq 1$. V is a positive potential and f is a critical nonlinearity with exponential growth. We derive a positive ground state solution by using minimax techniques combined with the fractional Trudinger–Moser inequality. Moreover, in the particular case of $a=1$ and $b=0$, we also obtain the existence of the ground state solution to the fractional Schrödinger equation.

1. Introduction and main results

The nonlocal problems are important in many fields of science, notably continuum mechanics, phase transition phenomena, population dynamics, minimal surfaces and anomalous diffusion, as they are the typical outcome of stochastic stabilization of Lévy processes, see [23], [2], [4], [28] and the references therein. In recent years, fractional equations have been widely studied, motivated by

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2020 *Mathematics Subject Classification.* Primary: 35A15, 35R11; Secondary: 47G20.

Key words and phrases. Fractional Kirchhoff–Schrödinger; critical exponential growth; fractional Trudinger–Moser inequality; the ground state solution.

The first author was partially supported by the Tianjin postgraduate research and innovation project (2019YJSB041).