Topological **M**ethods in **N**onlinear **A**nalysis
Volume 57, No. 1, 2021, 221–242
DOI: 10.12775/TMNA.2020.022

SINGULAR REACTION DIFFUSION EQUATIONS WHERE A PARAMETER INFLUENCES THE REACTION TERM AND THE BOUNDARY CONDITIONS

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Abstract. We analyse positive solutions to the steady state reaction diffusion equation:

$$\begin{cases} -u'' = \lambda h(t) f(u) & \text{in } (0,1), \\ -du'(0) + \mu(\lambda) u(0) = 0, \\ u'(1) + \mu(\lambda) u(1) = 0, \end{cases}$$

where $\lambda > 0$ is a parameter, $d \geq 0$ is a constant, $f \in C^2([0,\infty), \mathbb{R})$ is an increasing function which is sublinear at infinity $\Big(\lim_{s\to\infty} f(s)/s = 0 \Big)$, $h \in C^1((0,1],(0,\infty))$ is a nonincreasing function with $h_1 := h(1) > 0$ and there exist constants $d_0 > 0$, $\alpha \in [0,1)$ such that $h(t) \leq d_0/t^\alpha$ for all $t \in (0,1]$, and $\mu \in C([0,\infty),[0,\infty))$ is an increasing function such that $\mu(0) \geq 0$. We consider three cases of f, namely, $f(0) = 0$, $f(0) > 0$ and $f(0) < 0$. We will discuss existence and multiplicity results via the method of sub-supersolutions. Further, we will establish uniqueness results for $\lambda \approx 0$ and $\lambda \gg 1$.

1. Introduction

In [10], the authors studied positive solutions to classes of nonlinear elliptic boundary value problems where a parameter λ was involved in the reaction term

2020 *Mathematics Subject Classification.* Primary: 34B08, 34B15; Secondary: 34B16, 34B18.

Key words and phrases. Boundary value problems; singular problems; positive solutions.