MULTIPLICITY AND CONCENTRATION RESULTS FOR A CLASS OF SINGULARLY PERTURBED CRITICAL QUASILINEAR SCHRÖDINGER EQUATION

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ABSTRACT. In this paper, we study a class of singularly perturbed critical quasilinear Schrödinger equation of the form

$$-\varepsilon^2 \Delta u + V(x)u - \varepsilon^2(\Delta u^2)u = P(x)|u|^{p-2}u + Q(x)|u|^{2 \cdot 2^*-2}u, \quad \text{in } \mathbb{R}^N.$$

By using a change of variables and variational argument, we prove not only the existence of positive ground state solutions and their concentration behavior, but also the existence and associated concentration behavior of multiple solutions.

1. Introduction

Consider the following critical quasilinear Schrödinger equation

$$(1.1) \quad -\varepsilon^2 \Delta u + V(x)u - \varepsilon^2(\Delta u^2)u = P(x)|u|^{p-2}u + Q(x)|u|^{2 \cdot 2^*-2}u, \quad \text{in } \mathbb{R}^N,$$

where $N \geq 3$, $\varepsilon > 0$ is a parameter, $\max\{4, 2 \cdot (2^*-1)\} < p < 2 \cdot 2^*$, and 2^* is the critical Sobolev exponent. This type of equations arise when we are looking for standing waves $z(t,x) = \exp(-iEt)u(t)$ for the quasilinear Schrödinger equation

$$(1.2) \quad i\varepsilon \partial_t z = -\varepsilon^2 \Delta z + W(x)z - h(z) - \varepsilon^2 \Delta(|z|^2)z,$$