

EQUIVALENT FORMS OF THE BROUWER FIXED POINT THEOREM II

ADAM IDZIK — WŁADYSŁAW KULPA — PIOTR MAĆKOWIAK

ABSTRACT. Equivalents of the Brouwer fixed point theorem are proved. They involve formulations either for the standard simplex or for the cube. Characterizations of continuous functions defined on the standard simplex are also presented. The famous Steinhaus chessboard theorem is generalized.

1. Introduction

During the last 100 years, the classical Brouwer theorem on the existence of a fixed point for every continuous map defined on some compact convex set of a Euclidean space into itself has been generalized and applied in many directions [13].

In this paper we extend characterizations of the Brouwer theorem presented in [7]. In Section 3 we provide some equivalent versions of the labeled Sperner lemma and related results due to Bapat [2] and Gale [5]. In Section 4 we prove that an indexed closed (open) cover theorem, the Eilenberg–Otto theorem, the Poincaré theorem and the Bohl–Brouwer theorem are equivalent. Next, we present properties of continuous functions defined on the standard closed simplex. We also prove a lemma on the collapse [9]. Finally, we generalize the Steinhaus chessboard theorem [16] and the Gale theorem on hexagonal tiling [4], for an arbitrary finite tiling of the square.

2020 *Mathematics Subject Classification.* 54H25, 54H99.

Key words and phrases. Brouwer fixed point theorem; Steinhaus chessboard problem.