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ZERO TEMPERATURE LIMITS OF EQUILIBRIUM STATES FOR SUBADDITIVE POTENTIALS AND APPROXIMATION OF MAXIMAL LYAPUNOV EXPONENT

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Abstract. In this paper we study ergodic optimization problems for subadditive sequences of functions on a topological dynamical system. We prove that for $t \to \infty$ any accumulation point of a family of equilibrium states is a maximizing measure. We show that the Lyapunov exponent and entropy of equilibrium states converge in the limit $t \to \infty$ to the maximum Lyapunov exponent and entropy of maximizing measures. In the particular case of matrix cocycles we prove that the maximal Lyapunov exponent can be approximated by Lyapunov exponents of periodic trajectories under certain assumptions.

1. Introduction and statement of the results

Throughout this paper X is a compact metric space that is endowed with the metric d. We call (X, T) a *topological dynamical system* (TDS), if $T\colon X \to X$ is a continuous map on the compact metric space X. We say that $\Phi := \{\log \phi_n\}_{n=1}^{\infty}$ is a *subadditive potential* if each ϕ_n is a continuous non-negative-valued function on X such that

$$0 \le \phi_{n+m}(x) \le \phi_n(x)\phi_m(T^n(x)) \quad \text{for all } x \in X,\ m, n \in \mathbb{N}.$$

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