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BASIC RESULTS OF FRACTIONAL ORLICZ–SOBOLEV SPACE AND APPLICATIONS TO NON-LOCAL PROBLEMS

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Abstract. In this paper, we study the interplay between the Orlicz–Sobolev spaces L^M and $W^{1,M}$ and the fractional Sobolev spaces $W^{s,p}$. More precisely, we give some qualitative properties of a new fractional Orlicz–Sobolev space $W^{s,M}$, where $s \in (0,1)$ and M is a Young function. We also study a related non-local operator, which is a fractional version of the nonhomogeneous M-Laplace operator. As an application, we prove existence of a weak solution for a non-local problem involving the new fractional M-Laplacian operator.

1. Introduction

Recently, great attention has been focused on the study of fractional and non-local operators of elliptic type, both for in a purely mathematical research and in view of the concrete real-world applications. In most of these applications a fundamental tool to treat these type of problems are the so-called fractional order Sobolev spaces defined for $0 < s < 1 \leq p < \infty$ by

$$W^{s,p}(\Omega) = \left\{ u \in L^p(\Omega) : \frac{u(x) - u(y)}{|x-y|^{N/p+s}} \in L^p(\Omega \times \Omega) \right\},$$

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