Topological **M**ethods in **N**onlinear **A**nalysis
Volume 55, No. 2, 2020, 655–680
DOI: 10.12775/TMNA.2019.109

DOUBLE RESONANCE IN STURM–LIOUVILLE PLANAR BOUNDARY VALUE PROBLEMS

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Abstract. We provide some existence results for Sturm–Liouville boundary value problems associated with the planar differential system $Jz' = g(t,z) + r(t,z)$ where g is suitably controlled by the gradient of two positively homogeneous functions of degree 2 and r is sublinear with respect to the variable z at infinity. We study the existence of solutions when a double resonance phenomenon occurs by the introduction of Landesman–Lazer type conditions. Applications to scalar second order differential equations are given.

1. Introduction

For the scalar equation

$$x'' + f(t,x) = 0 \tag{1.1}$$

with periodic, Neumann or Dirichlet boundary conditions there have been several works concerning the existence of solutions under some nonresonance conditions.

The approach to resonance is a delicate problem and the most successful condition has been introduced by Landesman and Lazer, where the nonlinearity asymptotically lies between two eigenvalues of the linear differential equation, see [9], [10]. In the case of asymmetric nonlinearities we mention [6], [8] for

2020 *Mathematics Subject Classification.* 34B15, 37E45, 34A34.
Key words and phrases. Positively homogeneous planar systems; Sturm–Liouville boundary value problems; Dirichlet problem; shooting method; double resonance; Landesman–Lazer conditions.