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REGULARITY OF WEAK SOLUTIONS FOR A CLASS OF ELLIPTIC PDES IN ORLICZ–SOBOLEV SPACES

Jakub Maksymiuk — Karol Wroński

Abstract. We consider the elliptic partial differential equation in the divergence form

$$-\mathrm{div}(\nabla G(\nabla u(x))) + F_u(x, u(x)) = 0,$$

where G is a convex, anisotropic function satisfying certain growth and ellipticity conditions. We prove that weak solutions in $W^{1,G}$ are in fact of class $W^{2,2}_{\mathrm{loc}} \cap W^{1,\infty}_{\mathrm{loc}}$.

1. Introduction

We consider a quasilinear elliptic equation in the divergence form:

$$\text{(P)} \qquad -\mathrm{div}(\nabla G(\nabla u(x))) + F_u(x, u(x)) = 0$$

where $u\colon \Omega \to \mathbb{R}$ and $\Omega \subset \mathbb{R}^n$, $n \geq 1$ is an open connected set. Functions $G \in C^2(\mathbb{R}^n, \mathbb{R})$ and $F \in C^1(\Omega \times \mathbb{R}, R)$ are assumed to satisfy certain growth conditions given below. The objective of this paper is to show that for such G and F, every weak solution u, that belongs to the Orlicz–Sobolev space $W^{1,G}_{\mathrm{loc}}(\Omega)$, is of a class $W^{2,2}_{\mathrm{loc}}(\Omega) \cap W^{1,\infty}_{\mathrm{loc}}(\Omega)$.

This result is inspired by Marcellini's articles [14] and [13] in which he proves analogous regularity theorem for weak solutions of an elliptic equation. One of the differences between our result and these by Marcellini is that we assume

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