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BORSUK–ULAM THEOREMS FOR PRODUCTS OF SPHERES AND STIEFEL MANIFOLDS REVISITED

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Abstract. We give a different and possibly more accessible proof of a general Borsuk–Ulam theorem for a product of spheres, originally due to Ramos. That is, we show the non-existence of certain $(\mathbb{Z}/2)^k$-equivariant maps from a product of k spheres to the unit sphere in a real $(\mathbb{Z}/2)^k$-representation of the same dimension. Our proof method allows us to derive Borsuk–Ulam theorems for certain equivariant maps from Stiefel manifolds, from the corresponding results about products of spheres, leading to alternative proofs and extensions of some results of Fadell and Husseini.

1. Introduction

Let X be a compact n-dimensional CW complex with an action by the group G. A fundamental question with a multitude of applications in topological combinatorics is to decide whether an equivariant map $X \to V$ (that is, a map commuting with a G-action) into some n-dimensional real G-representation V must have $0 \in V$ in its image. Equivalently, one is interested in deciding the existence of an equivariant map $X \to S(V)$ into the unit sphere of V. This method has found applications in hyperplane mass partitions [2], the "square-peg" problem [14], Tverberg-type results [6], and chromatic numbers of hypergraphs [13], among others; see [12], [20]. Thus the identification of easily computable obstructions to the existence of such equivariant maps is of fundamental importance.

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