ON CRITICAL PSEUDO-RELATIVISTIC HARTREE EQUATION WITH POTENTIAL WELL

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Abstract. The aim of this paper is to investigate the existence and asymptotic behavior of the solutions for the critical pseudo-relativistic Hartree equation

$$\sqrt{-\Delta + m^2} u + (\beta V(x) - \lambda) u = \left( \int_{\mathbb{R}^N} \frac{|u(z)|^{2^*_\mu}}{|x-z|^\mu} \, dz \right) |u|^{2^*_\mu - 2} u$$

for $\mathbb{R}^N$, where $m, \lambda, \beta \in \mathbb{R}^+$, $0 < \mu < N$, $N \geq 3$, $2^*_\mu = (2N - \mu)/(N - 1)$ plays the role of critical exponent due to the Hardy–Littlewood–Sobolev inequality. By transforming the nonlocal problem into a local one via the Dirichlet-to-Neumann map, we are able to obtain the existence of the solutions by variational methods. Suppose that $0 < \lambda < \lambda_1(\Omega)$ with $\lambda_1(\Omega)$ the first eigenvalue and the parameter $\beta$ is large enough, we can prove the existence of ground state solutions. Furthermore, for any sequences $\beta_n \to \infty$, we can show that the ground state solutions $\{u_n\}$ converges to a solution of

$$\sqrt{-\Delta + m^2} u - \lambda u = \left( \int_{\Omega} \frac{|u(z)|^{2^*_\mu}}{|x-z|^\mu} \, dz \right) |u|^{2^*_\mu - 2} u \quad \text{in } \Omega,$$

where $\Omega := \text{int } V^{-1}(0)$ is a nonempty bounded set with smooth boundary. By the way we also establish the existence and nonexistence results for the ground state solutions of the problems set on bounded domain.

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