Topological **M**ethods in **N**onlinear **A**nalysis
Volume 54, No. 2B, 2019, 1093–1110
DOI: 10.12775/TMNA.2019.089

© 2019 Juliusz Schauder Centre for Nonlinear Studies
Nicolaus Copernicus University in Toruń

CONVERGENCE ESTIMATES FOR ABSTRACT SECOND ORDER DIFFERENTIAL EQUATIONS WITH TWO SMALL PARAMETERS AND MONOTONE NONLINEARITIES

Andrei Perjan — Galina Rusu

Abstract. In a real Hilbert space H we consider the following perturbed Cauchy problem

$$(\mathrm{P}_{\varepsilon\delta}) \quad \begin{cases} \varepsilon\, u''_{\varepsilon\delta}(t) + \delta\, u'_{\varepsilon\delta}(t) + Au_{\varepsilon\delta}(t) + B(u_{\varepsilon\delta}(t)) = f(t), \quad t \in (0,T), \\ u_{\varepsilon\delta}(0) = u_0, \quad u'_{\varepsilon\delta}(0) = u_1, \end{cases}$$

where $u_0, u_1 \in H$, $f\colon [0,T] \mapsto H$ and ε, δ are two small parameters, A is a linear self-adjoint operator, B is a locally Lipschitz and monotone operator. We study the behavior of solutions $u_{\varepsilon\delta}$ to the problem $(\mathrm{P}_{\varepsilon\delta})$ in two different cases:

(i) when $\varepsilon \to 0$ and $\delta \geq \delta_0 > 0$;
(ii) when $\varepsilon \to 0$ and $\delta \to 0$.

We obtain some *a priori* estimates of solutions to the perturbed problem, which are uniform with respect to parameters, and a relationship between solutions to both problems. We establish that the solution to the unperturbed problem has a singular behavior, relative to the parameters, in the neighborhood of $t = 0$. We show the boundary layer and boundary layer function in both cases.

2010 *Mathematics Subject Classification.* 35B25, 35K15, 35L15, 34G10.

Key words and phrases. Singular perturbation; abstract second order Cauchy problem; boundary layer function; a priori estimate.

Researches supported by the Program 15.817.02.26F.