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A PERIODIC BIFURCATION PROBLEM DEPENDING ON A RANDOM VARIABLE

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Dedicated to the memory of Professor Ioan I. Vrabie

Abstract. We consider an abstract bifurcation equation $P(x) + \varepsilon Q(x, \varepsilon, \omega) = 0$, where P and Q are operators, ε is the bifurcation parameter, $\omega \in \Omega$, is the random variable and $(\Omega, \mathcal{F})$ is a measurable space. The aim of the paper is to provide conditions on P and Q to ensure the existence, for any $\omega \in \Omega$, of a branch of solutions originating from the zeros of the operator P. We show that the considered abstract bifurcation is the model of a random autonomous periodically perturbed differential equation having the property that the unperturbed equation corresponding to $\varepsilon = 0$ has a limit cycle. As a consequence we obtain the existence, for any $\omega \in \Omega$, of a branch of periodic solutions of the perturbed equation emanating from the limit cycle.

1. Introduction

In this paper we consider the bifurcation equation of the form

$$P(x) + \varepsilon Q(x, \varepsilon, \omega) = 0, \tag{1.1}$$

where $P\colon \mathbb{E} \mapsto \mathbb{E}$ and $Q\colon \mathbb{E} \times [0,1] \times \Omega \mapsto \mathbb{E}$ are operators, $\mathbb{E}$ is a separable Banach space, $\varepsilon \geq 0$ is the bifurcation parameter, $\omega \in \Omega$ is the random variable and

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