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SUMS OF CONVEX COMPACTA AS ATTRACTORS OF HYPERBOLIC IFS'S

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Dedicated to the memory of Professor Ioan I. Vrabie

Abstract. We prove that a finite union of convex compacta in $\mathbb{R}^n$ may be represented as the attractor of a hyperbolic IFS. If such a union is the condensation set for some hyperbolic IFS with condensation, then its attractor can be represented as the attractor of a standard hyperbolic IFS. We illustrate this result with the hyperbolic IFS with condensation, whose attractor is the well-known "The Pythagoras tree" fractal.

1. Introduction

The term *fractal* is usually associated with the attractor of a hyperbolic *Iterated Function System* (see, e.g. [3]). The main ingredients of fractals are self-similarity and fractal dimension.

M. Barnsley [3] has introduced the idea of an *Iterated Function System with condensation*, which means a hyperbolic IFS, accompanied by a constant compact-valued multi-function (condensation). This idea has led to new fractals as attractors of IFS's, mostly related to Cantor sets. However, the computer simulations of such IFS's create more problems than for hyperbolic IFS's.

M. Hata [18] showed that every connected attractor of a hyperbolic IFS must be locally connected. He asked whether there exists a locally connected compact set, which is not attractor of any IFS.

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