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EXISTENCE RESULTS FOR EVOLUTION EQUATIONS WITH SUPERLINEAR GROWTH

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Dedicated to the memory of Professor Ioan I. Vrabie

Abstract. By combining an approximation technique with the Leray–Schauder continuation principle, we prove global existence results for semilinear differential equations involving a dissipative linear operator, generating an extendable compact C_0-semigroup of contractions, and a Carathéodory nonlinearity $f\colon [0,T]\times E\to F$, with E and F two real Banach spaces such that $E\subseteq F$, besides imposing other conditions. The case $E\neq F$ allows to treat, as an application, parabolic equations with continuous superlinear nonlinearities which satisfy a sign condition.

1. Introduction

Let $(E,\|\cdot\|_E)$, $(F,\|\cdot\|_F)$ be two real Banach spaces such that $E\subseteq F$. This work deals with the study of mild solutions for semilinear differential equations

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