

## A DUAL REPRESENTATION RESULT FOR VALUE FUNCTIONS IN STOCHASTIC CONTROL OF INFINITE DIMENSIONAL GROUPS

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*Dedicated to the memory of Professor Ioan Vrabie*

**ABSTRACT.** We study here the problem of dual representation of the value functions associated to linear-convex stochastic control problems in infinite dimensional Hilbert spaces. Since the dual state runs backwards in time, it turns out that the dual representation has the meaning of a classical (Markov) control problem only if the primal linear state equation is driven by the generator of a group. In the general case, a dual representation of the value function still holds, but such a representation cannot be reduced to solving a dual Hamilton–Jacobi–Bellman equation.

### 1. Introduction

It is well known since Bismut [3] that convex duality between spaces of martingales together with the Fenchel–Legendre transform of convex functions can be used as a powerful tool to solve stochastic control problems. This line of ideas has been used extensively, especially in the Mathematical Finance literature related to Optimal Investment. In this context, duality is not only used to get a representation of the value function in terms of its dual, but is actually an essential part in proving existence of optimizers in the primal problem. This is exactly the case in [5] and [6], where additional technical difficulties come from

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2010 *Mathematics Subject Classification.* Primary: 49N15, 93E20; Secondary: 93C05, 93C25.

*Key words and phrases.* Value function; Legendre transform; duality.

The second author was supported in part by NSF Grant DMS # 1517664.