Topological **M**ethods in **N**onlinear **A**nalysis
Volume 54, No. 2A, 2019, 833–862
DOI: 10.12775/TMNA.2019.076

TOPOLOGY OF TWISTS, EXTREMISING TWIST PATHS AND MULTIPLE SOLUTIONS TO THE NONLINEAR SYSTEM IN VARIATION $\mathscr{L}[u] = \nabla\mathscr{P}$

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Abstract. In this paper we address questions on the existence and multiplicity of a class of geometrically motivated mappings with certain symmetries that serve as solutions to the nonlinear system in variation:

$$\mathrm{ELS}[(u,\mathscr{P}),\Omega] = \begin{cases} [\nabla u]^t \mathrm{div}[F_\xi \nabla u] - F_s[\nabla u]^t u = \nabla\mathscr{P} & \text{in } \Omega, \\ \det\nabla u = 1 & \text{in } \Omega, \\ u \equiv x & \text{on } \partial\Omega. \end{cases}$$

Here $\Omega \subset \mathbb{R}^n$ is a bounded domain, $F = F(r, s, \xi)$ is a sufficiently smooth Lagrangian, $F_s = F_s(|x|, |u|^2, |\nabla u|^2)$ and $F_\xi = F_\xi(|x|, |u|^2, |\nabla u|^2)$ with F_s and F_ξ denoting the derivatives of F with respect to the second and third variables respectively while $\mathscr{P}$ is an *a priori* unknown hydrostatic pressure resulting from the incompressibility constraint $\det\nabla u = 1$. Among other things, by considering twist mappings u with an $\mathrm{SO}(n)$-valued twist path, we prove the existence of multiple and topologically distinct solutions to ELS for $n \geq 2$ even versus the only (*non*) twisting solution $u \equiv x$ for $n \geq 3$ odd. An extremality analysis for twist paths and those of Lie exponential types and a suitable formulation of a differential operator action on twists relating to ELS are the key ingredients in the proof.

2010 *Mathematics Subject Classification.* 35J57, 35J50, 35J62, 49J10, 35A15, 53C22, 58D19, 22E30.

Key words and phrases. Nonlinear elliptic systems; incompressible twists; multiple solutions; restricted energies; closed geodesics on $\mathrm{SO}(n)$; self-maps of spheres, Hopf degree.